

## CORRECTION EXAMEN SIGNAUX ET SYSTEMES

### Exercice 1 (pts) (4 pts)

- 1) expression mathématique:  $f(t) = \begin{cases} A & \text{si } 0 < t \leq \frac{T_0}{2} \\ -A & \text{si } \frac{T_0}{2} < t \leq T_0 \end{cases}$  1pt
- 2) Depuis le graphique  $f(t)$  est une fonction impaire: alors:  $A_0 = A_m = 0$  0 pt.
- $$B_m = \frac{4}{T_0} \int_{-T_0/2}^{T_0/2} f(t) \sin(m\omega t) dt = \frac{2A}{\pi} \int_0^{\pi} 1 \cdot \sin(m\omega t) dt = -\frac{2A}{m\pi} [\cos(m\omega t)]_0^{\pi}$$
- $$B_m = -\frac{2A}{m\pi} (\cos(m\pi) - 1) = \frac{2A}{m\pi} (1 - \cos(m\pi))$$
- Si:  $\left\{ \begin{array}{l} m \text{ est paire: } B_m = 0 \\ m \text{ est impaire: } B_{2m+1} = \frac{4A}{m\pi} \end{array} \right.$  2 pts
- Exercice 2 (pts) (4 pts)**

$$f(t) = \sum_{m=0}^{+\infty} \frac{4A}{(2m+1)\pi} \sin((2m+1)\omega t)$$

$$\text{TF}[\text{sgn}(t) e^{-at}] = \int_{-\infty}^0 -e^{at} e^{-j\omega t} dt + \int_0^{+\infty} e^{-at} e^{-j\omega t} dt$$

$$\text{TF}[\text{sgn}(t) e^{-at}] = \frac{-1}{a - j\omega} + \frac{1}{a + j\omega} = \frac{-2j\omega}{a^2 + \omega^2}$$

Donc:

$$\text{TF}[\text{sgn}(t) e^{-at}] = \frac{-2j\omega}{a^2 + \omega^2} \quad \text{si } \text{Re}(a) > 0$$
2 pts

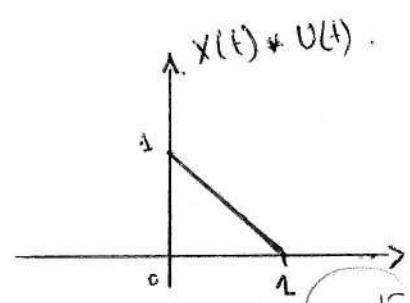
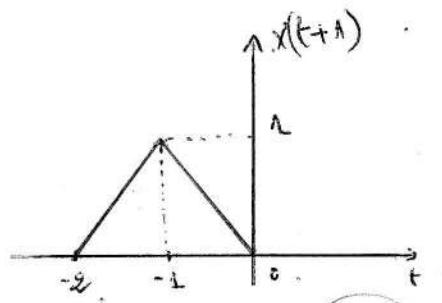
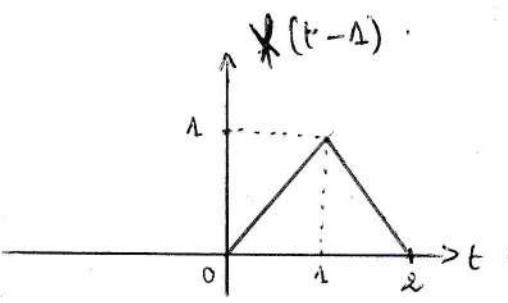
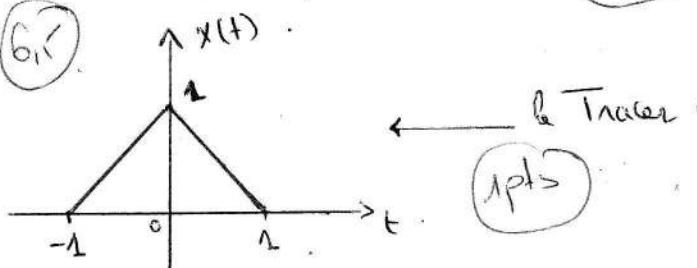
1. Calcule de amplitude et angle.

$$|x(f)| = x = \frac{4\pi f}{\sqrt{a^2 + 4\pi^2 f^2}}$$
1 pt

$$\Theta = \arg(x(f)) = \arctg\left(\frac{-2\pi f}{a^2 - 4\pi^2 f^2}/0\right) = \arctg(\infty)$$
1 pt

### Exercice 3 (pts) (6 pts)

A- :



$x(t-1)$

$x(t+1)$

$x(t) * u(t)$

B-

$$F(w) = \sum_{n=-\infty}^{\infty} 2^{-n} U(-n) e^{-jwn} \quad \underline{F(w) = \sum_{n=-\infty}^0 2^{-n} e^{-jwn}}$$

On fait un changement de variable on pose  $n=-l$  donc  $l=-n$

$$F(w) = \sum_{l=0}^{\infty} 2^l e^{jwl}$$

$$F(w) = \sum_{l=0}^{\infty} (2^1 e^{jw})^l \quad \underline{F(w) = \sum_{l=0}^{\infty} (2 e^{jw})^l}$$

2,5 pts

$$F(w) = \frac{1}{1 - 2 e^{jw}}$$

Exercice 4 (pts)

5,5 pts

$$1- x(t) = 2A \cos(2\pi f_0 t) \quad \text{avec} \quad f_0 = 2 \text{ hz}$$

$$\underline{x(f) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt = \int_{-\infty}^{\infty} 2A \cos(2\pi f_0 t) e^{-jwt} dt}$$

On sait que  $\int_{-\infty}^{\infty} e^{-jwt} dt = \delta(f)$

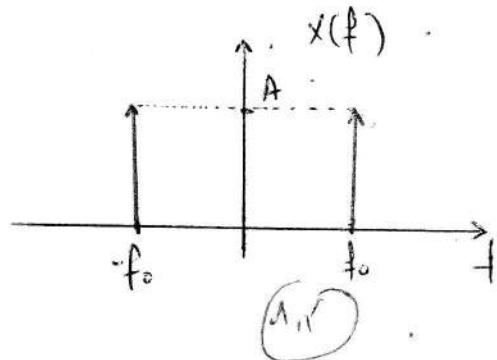
A<sub>1</sub>?

$$x(f) = A[\delta(f-f_0) + \delta(f+f_0)]$$

2- condition de Shannon

$f_e > 2f_m$  : reconstitution possible du signal

$f_e < 2f_m$  : reconstitution impossible du signal



$$f_e = 1/t_e \text{ donc } 1/t_e > 2f_m \text{ donc } \underline{1/t_e > 2*2} \quad 1/t_e > 4$$

$$\text{alors } t_e < \frac{1}{4} \quad t_e < 0.25$$

A<sub>2</sub> pts

3- si  $t_e = 0.2 \quad 0.2 < 0.25$  ~~reconstitution parfaite du signal analogique~~  
 si  $t_e = 0.5 \quad 0.5 > 0.25$  ~~reconstitution impossible du signal analogique~~

1,5 pts