Written exam (correction)

First mane:................................. Last mane :.................................

Choose the correct answer:

1) Let the random process defined by: $X(t) = A\cos(\omega_0 t + \Theta)$, where, A and ω_0 are constants, Θ is a random variable with a probability density function:

$$f_\Theta(\theta) = \begin{cases} \frac{4}{\pi}, & |\theta| \le \frac{\pi}{8} \\ 0, & \text{otherwise} \end{cases}$$

a) Its mathematical expectation is equal to:

- [x] $\frac{4\sqrt{2}A}{\pi}\cos\omega_0 t$
- [ ] A
- [ ] $\frac{\sqrt{2}A}{\pi}$
- [ ] $\frac{4\sqrt{2}A}{\pi}$

b) Its autocorrelation function equals:

- [ ] $\frac{A^2}{2}\cos\omega_0\tau$
- [x] $\frac{A^2}{2}\cos\omega_0\tau + \frac{2A^2}{\pi}\cos(2\omega_0 t + 2\omega_0\tau)$
- [ ] $A^2\cos(2\omega_0 t + 2\omega_0\tau)$
- [ ] $\frac{2A^2}{\pi}\cos(2\omega_0 t + 2\omega_0\tau)$

c) The random process $X(t)$ is :

- [x] Not stationary and not ergodic
- [ ] Strict-Sense Stationary
- [ ] Stationary and ergodic
- [ ] Wide-Sense Stationary

2) Consider the hypothesis testing problem in which:

$$f_{Y|H_0}(y|H_0) = \text{rect}\left(y - \frac{1}{2}\right) \quad \text{and} \quad f_{Y|H_1}(y|H_1) = \frac{1}{2}\text{rect}\left(\frac{y-1}{2}\right)$$

a) $\eta > 1/2$, the decision regions are :

- [ ] For $0 \le y \le 1 \Rightarrow$ decide H_1 and for $1 \le y \le 2 \Rightarrow$ decide H_0
- [ ] decide H_1 or H_0 in at the range $0 \le y \le 1$ and decide H_1 for $1 < y \le 2$.
- [ ] always decide H_1
- [x] For $0 \le y \le 1 \Rightarrow$ decide H_0 and for $1 \le y \le 2 \Rightarrow$ decide H_1

b) The probability of false alarm is :

- [ ] $P_F = \int_{Z_1} f_{Y|H_0}(y|H_0)dy = \int_0^1 0\,dy = 0$
- [x] $P_F = \int_{Z_1} f_{Y|H_0}(y|H_0)dy = \int_1^2 0\,dy = 0$
- [ ] $P_F = \int_{Z_1} f_{Y|H_0}(y|H_0)dy = \int_1^2 1\,dy = 1$
- [ ] $P_F = \int_{Z_1} f_{Y|H_0}(y|H_0)dy = \int_0^1 1\,dy = 1$

3) Let $X_1, \ldots, X_n$ be independent and identically distributed random variables whose density is defined by:

$$f_\theta(x) = \begin{cases} \frac{x}{\theta}\exp\left(-\frac{x^2}{2\theta}\right) & \text{si } x > 0 \\ 0 & \text{Otherwise} \end{cases}$$

a) The MLE of θ is given by:

☐ $\hat{\theta}_{MLE} = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ ☐ $\hat{\theta}_{MLE} = \frac{1}{n}\sum_{i=1}^{n} x_i$ ☒ $\hat{\theta}_{MLE} = \frac{1}{2n}\sum_{i=1}^{n} x_i^2$ ☐ $\hat{\theta}_{MLE} = \frac{1}{2n}\sum_{i=1}^{n} x_i$

b) The Cramer-Rao Bound is as follows :

☐ $\text{BCR} = \frac{\theta^2}{n}$ ☐ $\text{BCR} = \frac{\theta}{2n}$ ☒ $\text{BCR} = \frac{\theta^2}{2n}$ ☐ $\text{BCR} = \frac{\theta}{n}$

4) The Doppler shift can be calculated using the expression: $\delta f = f_c \frac{2v\cos\theta}{c}$.

a) If f_c=25GHz, $c=3\times10^8$ m/s, $\theta=25°$ and δf=3.3567 KHz, so:

☒ v=80 Km/h ☐ v=20 m/s ☐ v=25m/s ☐ v=120Km/h

b) If f_c=25GHz, $c=3\times10^8$ m/s, v=100Km/h and δf=4.3504 KHz, so:

☐ $\theta=30°$ ☐ $\theta=15°$ ☒ $\theta=20°$ ☐ $\theta=25°$

5) Using 3 distributed CA-CFAR detectors (identical case), the probability of false alarm, using exponential conditional probability laws and the "And" fusion rule is given by the following expression:

$$P_F = [1+T]^{-3N}$$

a) If N=32 and $P_F=10^{-4}$, the constant multiplier T equals:

☐ 0.3335 ☐ 0.7783 ☐ 0.2115 ☒ 0.1007

b) By considering these conditions and setting μ=25 dB, the detection probability is equal to :

☐ $P_D = \left[1+\frac{T}{(1+\mu)}\right]^{-3N} = 0.9685$ ☐ $P_D = \left[1+\frac{T}{(1+\mu)}\right]^{3N} = 0.69$

☒ $P_D = \left[1+\frac{T}{(1+\mu)}\right]^{-3N} = 0,97$ ☐ $P_D = \left[1+\frac{T}{(1+\mu)}\right]^{N} = 0.9669$

Good luck